Using Computer Simulation Methods to Teach Statistics: A Review of the Literature

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Abstract

The teaching and learning of statistics has impacted the curriculum in elementary, secondary, and post-secondary education. Because of this growing movement to expand and include statistics into all levels of education, there is also a considerable interest in employing effective instructional methods, especially for statistics concepts that tend to be very difficult or abstract. Researchers have recommended using computer simulation methods (CSMs) to teach these concepts; however, a review of the literature reveals very little empirical research to support the recommendations. The purpose of this paper is to summarize and critically evaluate the literature on how CSMs are used in the statistics classroom and their potential impact on student achievement. The recommendation is that more empirically and theoretically grounded research studies are needed to determine if these methods improve student learning.

1. Introduction

The teaching and learning of statistics has pervaded all levels of education and has gained recognition in many disciplines over the past two decades. Statistics continues to be an integral part of the post secondary curriculum. In almost every discipline, the ability to understand, interpret, and critically evaluate research findings is becoming an essential core skill (Giesbrecht 1996). Buche and Glover (1988) agree in that college students interested in becoming practitioners need to be able to comprehend, appreciate, and apply research.

In recent years, however, an appreciation of the importance of statistics in the elementary and secondary grades has also evolved. There is a growing movement to introduce concepts of statistics and probability into the elementary and secondary school curriculum. The implementation of the Quantitative Literacy Project (QLP) of the American Statistical Association (ASA) is one indication of interest in this movement (Scheaffer 1988). The QLP provides instructional materials on probability and statistics that can be used in the pre-college curriculum. In addition, the release of the NCTM *Principles and Standards for School Mathematics* (NCTM 2000), designed to improve mathematics education from pre-

kindergarten to grade 12, includes a content standard that also emphasizes statistical reasoning ("Data Analysis and Probability"). Consequently, many states now include and emphasize statistical thinking in their statewide curriculum guidelines. Because of this growing movement to expand and include statistics into all curricula, Becker (1996) stated that there is also considerable interest in **how** to teach statistics, in a variety of fields (Richardson 1991) and to a variety of age groups (Shulte 1979).

Another important change that has had a major impact on the teaching and learning of statistics over the past few decades has been the integration of computers, particularly in the statistics post-secondary classrooms. Microcomputer development has led to increased accessibility for students and an increase in the development of more user-friendly statistics packages (consider SAS®, SPSS®, Excel®, and MINITAB® as examples). An advantage of the microcomputer in the statistics classroom is that it allows students to accomplish computational tasks more quickly and efficiently, thereby freeing them to focus more on statistics concepts. Therefore, the computer not only operates as a powerful computational tool, but it can also help to reinforce specific concepts by providing settings in which students can apply statistics concepts and techniques.

Students actively involved in analyzing data using statistical software have obtained a more thorough understanding of statistics concepts (see Goodman 1986; Hubbard 1992; Mittag 1992; Gratz, Volpe, and Kind 1993; Packard, Holmes, and Fortune 1993; Sullivan 1993; Giesbrecht 1996; Marasinghe, Meeker, Cook, and Shin 1996; McBride 1996; Velleman and Moore 1996). With this increasing use of technology, however, additional research and discussion is needed on the appropriate ways to use computers in the statistics classroom to **specifically** identify and determine their effect on student learning.

The microcomputer not only has encouraging educational advantages for students, but it also continues to play an important role in statistics instruction. The statistics curriculum and the microcomputer together offer many opportunities to help instructors achieve their own pedagogical objectives. The majority of college instructors use statistics software primarily for students to perform routine data analysis tasks, often in hopes of enhancing student learning. These assignments enable most students to master the **mechanics** of data analysis (Marasinghe et al. 1996) but even when students use software packages to apply techniques, **abstract** statistics concepts may still be difficult for students to comprehend.

One exciting advantage of the microcomputer, which has been suggested in the literature, lies in its capability of enhancing student understanding of abstract or difficult concepts (Kersten 1983; Dambolena 1986; Gordon and Gordon 1989; Shibli 1990; Kalsbeek 1996; Hesterberg 1998; and many others). By using current computing technology, it is possible to supplement standard data analysis assignments by providing students with additional statistical experiences through the use of computer simulation methods (CSMs). CSMs allow students to experiment with random samples from a population with known parameters for the purpose of clarifying abstract and difficult concepts and theorems of statistics. For example, students can generate 50 random samples of size 30 from a non-normal distribution and compute the mean for each random sample. A histogram of the sample means can show the student that the sampling distribution of the sample mean is normally distributed. Computer simulations are invaluable in this regard because hard to understand concepts can be illustrated visually using many of the standard packages used to compute statistics (such as Excel® and MINITAB®). This can enhance the learning experience, especially for students in introductory statistics courses.

1.1 Purpose

Many researchers have recommended using CSMs to help teach statistics concepts, particularly for difficult or abstract concepts. The primary purpose of this paper is to review the literature on how CSMs are used in the statistics classroom. Journal articles and studies related to these methods are summarized

and discussed for selected statistics topics and are evaluated in terms of their impact on student achievement and their usefulness in today's statistics classroom.

The next section presents a theory of learning known as constructivism, which provides one theoretical framework of how students can learn statistics. A brief overview of this theory is provided and its application to CSMs is considered for the articles reviewed.

1.2 Constructivism

Many researchers in education and psychology support the theory that students learn by actively building or constructing their own knowledge and making sense out of this knowledge. Individuals construct new knowledge internally by transforming, organizing, and reorganizing previous knowledge (Cobb 1994; Greeno, Collins, and Resnick 1996) as well as externally, through environmental and social factors that are influenced by culture, language, and interactions with others (Bruning, Schraw, and Ronning 1999). Constructivism suggests that new knowledge is not passively received from the teacher to the student through textbooks and lectures, or by simply asking students to memorize rote facts. Instead, meaning is acquired through a significant interaction with new knowledge (Von Glasersfeld 1987). Regardless of how clearly a teacher explains a concept, students will understand the material only after they have constructed their own meaning for the new concepts, which may require restructuring and reorganizing new knowledge and linking it to prior or previous knowledge (Von Glasersfeld 1987). Constructivism also suggests that learning should be facilitated by teachers and that interaction and discussion are critical components during the learning process (Eggen and Kauchak 2001).

The application of learning statistics using CSMs may benefit students by empowering them to develop their own understanding of statistics concepts. Students will have the opportunity to learn by constructing their own ideas and knowledge from the computer simulation experiences, with supportive direction from the instructor. According to Packard et al. (1993), students who are actively involved in their own learning usually become more independent learners and problem solvers.

1.3 Definition of Computer Simulation Methods

Four definitions of computer simulation methods were described in the literature reviewed for this article. One definition involved students writing their own programs (using SAS PROC IML®, say), setting up a model for a problem and investigating diagnostics for the model in seeking possible violations of assumptions. A second definition allowed students to experience similar advantages using a random number generator in Excel® or MINITAB®. Using Excel® or MINITAB®, the commands to generate the random samples and perform experiments on the model are mostly window-driven. Third, many instructors used some combination of the first and second definitions, by providing program templates that allowed students to change parameters during the experiments (commonly in SAS® or SPSS®). Finally, a fourth definition involved using commercial software packages designed exclusively for simulation purposes (for example, the "Samplings Distribution" program). The literature reviewed in this paper included journal articles that utilized all four operational definitions, although the majority of the authors reported using the latter three.

Although this paper focuses on the use of computer simulation methods, it is also important to mention that a few of the articles reviewed also utilized physical or manual simulations before or after using CSMs. Physical simulations may involve exercises and may use devices such as coins, dice, or other objects that may be manipulated by the students and teacher to further clarify difficult statistics concepts. Some authors pointed out the academic benefits of preparing an "advanced organizer" or motivating students through the use of real-life statistical problems before using CSMs, especially since "random number generators can mystify beginning students" (Kaigh 1996, p.87). Although this paper does not

consider or review the effects of physical simulations, the general consensus from the review revealed that both physical and computer simulation exercises appear to complement one another and either or both are effective classroom strategies to enhance student comprehension.

Below is a review and critical examination of the literature related to the teaching and learning of statistics using CSMs, primarily considering students in an introductory statistics course in the post secondary arena. Various statistics topics have been suggested by many researchers, including topics for introductory to advanced statistics courses. References within each topic include descriptions by authors who have utilized particular methods or are simply reporting the potential benefit of CSMs for the topic of interest. The articles are reviewed in this manner due to the fact that for the topic of interest (consider, for example, the Central Limit Theorem), nearly identical simulation techniques were used across studies. A discussion of the type of simulation method used, evidence of a theory of learning, and other comments specific to the article is also discussed. Concluding remarks are also provided.

2. Literature Review

The first literature search was conducted from 1983 to 2000 using the following databases: Business and Economics, including Econlit and Periodical Abstracts; Education, including Educational Resources Information Center (ERIC) database, Social Science Abstracts, Current Contents, and Educational Abstracts; General Indexes, including Dissertation Abstracts; and Social Sciences, including PsycINFO and SociABS. There were 178 references using keywords "simulation" and "statistics." Eighteen of these studies were strictly related to the teaching and learning of statistics using CSMs and only one study was empirically based. A second literature search was also conducted using the *Current Index to Statistics* (*CIS*) printed volumes from 1990-1998 and Version 2.0 (Release 9) of the *CIS* CD-ROM extended database. The following keywords were considered: "computer program or experiment," "computing or computing environment," "education," "empirical process or research," "simulation," "software," "statistics," "teaching," "Web simulation," "World Wide Web simulation," and "Internet computer simulation." Approximately 88 articles were identified, including many articles that had already been identified in the first search. Of these 88 articles, 24 were appropriate for this review and one study was empirically-based. The remaining articles from both searches involved other computer or simulation-related ideas pertaining to statistics education but were not appropriate for this review. Some of the ideas represented in this latter group included physical simulation, using the computer to teach statistics, and discussions of computer programs available. Finally, the *Journal of Statistics Education* (*JSE*) archive 1993-2000 was also reviewed which resulted in six articles related to CSMs. One other empirical study was identified. A summary table of the 48 articles is provided in the Appendix.

From this review, it is clear that while many researchers in the field of statistics education recommend the use of CSMs to teach abstract concepts (see Kersten 1983; Dambolena 1986; Goodman 1986; Gordon and Gordon 1989; Shibli 1990; Prybutok, Bajgier, and Atkinson 1991), an examination of the related literature in this area revealed two important key issues. First, although many of the published articles advocated the use of simulation methods, only a very few were empirically based. If simulation methods are indeed helpful, why is it that researchers have not documented the empirical results to verify their suggestions? Second, even though some of the research in statistics education has been grounded in the theory of constructivism, very few of the journal articles related to CSMs specifically mentioned or identified a general theory of learning. Most of the articles, however, did involve students actively involved in their own learning and construction of knowledge. Consequently, elements of this theory were evident in the author's recommendations and the journals reviewed will be discussed considering a constructivist theory perspective. The Central Limit Theorem (CLT) was one of the more popular topics used with CSMs and will therefore begin the summary and discussion of the literature.

2.1 Central Limit Theorem

Interactive simulation programs on the World Wide Web (WWW) are the latest Internet resources many educators are now using to illustrate statistics concepts. Ng and Wong (1999) reported using simulation experiments on the Internet to illustrate Central Limit Theorem (CLT) concepts. At URL www.ruf.rice.edu/~lane/stat_sim/sampling_dist/index.html the CLT can be demonstrated graphically, either in large lectures or by the student with some guidance from the instructor. The program begins by allowing the user to choose a distribution from which the data are to be generated, a sample size for the sampling distribution for the mean, and the number of samples to be drawn. By changing the sample size, the user can observe how fast the probability histogram approaches the normal curve as the sample size increases. The program also allows the user to compare sampling distributions of other statistics as well such as the median and standard deviation. Many other statistics educators have used simulation exercises on the Internet for the CLT (see West and Ogden 1998) and with other topics (Schwarz 1997; Schwarz and Sutherland 1997).

Because most educators have access to the WWW, statistics simulation programs and other statistics resources have certainly provided an exciting new medium for teaching and learning. Using a browser such as Netscape Navigator® or Microsoft Internet Explorer® with Java® capabilities is the only user requirement. Although this newer technology has grown in popularity, simulation activities using other programs (MINITAB®, BASIC®, and SAS®, for example) were used predominantly during the last two decades. A discussion of these simulation techniques and how they have been used in the classroom (and still today) is also relevant.

According to Kersten (1983), simulation methods can clarify concepts and theorems of statistics (such as the CLT) and may also allow the non-mathematically oriented student in elementary statistics to have inductive experiences with statistical concepts in a very time-efficient manner. The MINITAB® command IRANDOM can be used to generate random numbers between 0 and 1, where each number is equally likely to occur (that is, $f(x) = 1$, $0 < x < 1$ for the uniform distribution). Three simulations can be performed, each of 300 random samples for $n = 1$, $n = 4$, and $n = 16$ from the IRANDOM population. The mean of each sample can be computed and the resulting means can be used to construct a histogram. The student can then see how the mean of the sample means is close to the population mean but the standard deviations decrease for the 300 random samples from $n = 1$ to $n = 16$. The student can also see from the histograms that as n becomes larger, the distribution of the sample means grows more similar to the normal distribution.

Dambolena (1986) agrees with others regarding the benefits of computer simulations to help beginning students in statistics develop a more intuitive understanding of the CLT. Using BASIC® programming, he suggested drawing a random sample of size 30 from a discrete uniform population with mean μ and standard deviation σ, computing the sample mean, and repeating this procedure 1000 times. Using MINITAB®, the 1000 means obtained from samples of size $n = 30$ can be output in a separate file that would subsequently be used to generate histograms and to illustrate the concepts of the CLT.

Many others have advocated the use of simulation methods to reinforce students' understanding of the concepts involving the CLT (see Pulley and Dolbear 1984; Gordon 1987; Karley 1990; Halley 1991; Bradley, Hemstreet, and Ziergenhagen 1992; Mittag 1992; Marasinghe et al. 1996; Hesterberg 1998; delMas, Garfield, and Chance 1999). Either a package such as MINITAB® or a statistics-simulation program may provide software flexibility for instructors to utilize these methods in the classroom. Software improvements in MINITAB® now eliminate the need for storing output in separate files for further analyses. In addition, having each student generate his or her own individual data provides the student with an experience that appears to be more convincing. Students are able to process their own ideas by building their own meanings through a significant interaction with the new information (Von Glasersfeld 1981).

Simulation experiences may allow students to gain a better understanding of the concept of an expected value, the shape of distributions of varying sizes, and the meaning of a sampling distribution. Using simulation methods to clarify these concepts early in the course can also aid as an advanced organizer for related statistics concepts taught later (such as sampling distributions for other statistics and their role in inferential statistics). On the other hand , according to Yu, Behrens, and Anthony (1995), interactive computer simulations to teach concepts of the CLT may further build upon other misconceptions, especially when students begin with unclear concepts of the properties of the CLT. Yu et al. (1995) concluded that some aspects of the CLT can be clearly illustrated by computer software but some cannot. For example, CSMs can perform the function of showing the process of a sampling distribution, but abstract concepts such as equality, independence, and the relationship between the CLT and hypothesis testing are difficult to present. The instructor should identify and explicitly address any misconceptions as well as any correct conceptions in addition to the use of CSMs.

The authors are at least partly justified in their concerns, in that just as there is the possibility of changing students' conceptions for deeper processing and comprehension of the properties related to the CLT, there is also the opportunity for students to misunderstand these concepts. In this case, students who are actively and independently involved in their own learning can "mis-construct" and further build upon misconceptions. Another disadvantage pointed out by the authors is that other abstract concepts related to the CLT such as independence, equality, and fairness cannot be illustrated in a computer environment. For example, it is unlikely that a student will find out that randomness will not guarantee equality and fairness after looking at the random sampling process on a computer screen many times (Yu et al. 1995).

Perhaps concepts such as independence and equality should not be taught in a computing environment. There are limitations associated with any teaching method, but the assumption that students use CSMs to construct their own meanings of CLT concepts does not imply or include the assumption that the teacher does not teach. Using these methods in conjunction with conventional pedagogy or other teaching strategies should obviously be evaluated.

2.2 t-Distribution

Dambolena (1986) suggested how to help introductory students develop a better understanding of concepts related to Student's *t*-distribution using MINITAB®. He advocated using simulation methods to verify that:

a. the distribution of $\sqrt{n}(\bar{x} - \mu)/s$ is centered around zero (where $\bar{x}$ is the sample mean, μ is the population mean, *s* is the sample standard deviation, and *n* is the sample size),

b. the distribution of $\sqrt{n}(\bar{x} - \mu)/\sigma$ should provide a smaller variance when σ, the population standard deviation, is known, and

c. this variance should decrease as the sample size increases.

A BASIC® program can be used to generate 1000 random samples of size *n* from the normal distribution with mean μ and standard deviation σ. The value of the *t*-statistic can be computed for each sample and these values can be saved to an output file (Again, later updates to the MINITAB® program will allow these two steps to be combined). Then, for different values of *n*, one can obtain relative frequencies and compare these with the probabilities from the appropriate *t*-distributions. Histograms can also be generated using different sample sizes to illustrate that as *n* increases, the histograms become more symmetric around their means (Dambolena 1986).

[Gordon and Gordon (1989)](#) also presented the use of a computer graphic simulation program to provide students with a means of transferring some of the rote learning in statistics of the *t*-distribution into a context of exploration and discovery. Gordon and Gordon suggest examining the sampling distribution of the difference in sample means. Samples of size n_1 and n_2 can be drawn from underlying populations with means μ_1 and μ_2 and standard deviations σ_1 and σ_2, respectively. The sampling distribution for the difference in sample means has mean $\mu_1 - \mu_2$ and the standard deviation (that is, standard error of the difference between sample means) is given by:

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

Because the population standard deviation is unknown, the sample standard deviations can be used in the estimated standard error of the difference between sample means. Independence between and within samples as well as normally distributed populations are assumed. If population variances are assumed to be equal, the estimated standard error is given by:

$$\sqrt{\left(\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$$

The user can select two populations (not necessarily normal or even identically distributed), the desired sample sizes to be drawn from each population, and the number of samples. The means can be calculated from each sample along with the differences in the sample means. The difference in means can be displayed in the form of a histogram with larger sample sizes illustrating a better approximation to the normal distribution. Also, the theoretical (population) values of the difference in means, as well as the theoretical standard deviations can be compared to the empirical mean and standard deviation for the sampling distribution of the difference in sample means.

Although [Dambolena (1986)](#) and [Gordon and Gordon (1989)](#) encouraged readers to use computer simulations and graphics to enhance students' understanding of the *t*-distribution, it is not readily apparent that these methods offer a better instructional method than a more traditional approach. Nevertheless, both exercises can be useful, particularly when exploration or discovery is of interest, as suggested. The recommendations to use these exercises for exploration purposes can provide students the opportunity to discover for themselves properties related to the *t*-distribution rather than simply accepting this information from a textbook. Constructivist ideas are again present in both authors' suggestions.

Two questions come to mind when considering [Damobolena's (1986)](#) computer simulation exercise. First, is this a worthwhile exercise that will facilitate students' subsequent learning of statistical concepts? Second, is this method necessary to clarify "abstract" properties related to the *t*-distribution? In light of time constraints which often affect today's statistics classrooms, simulation methods to verify probabilities or areas under the *t*-distribution may not be a priority for some instructors. On the other hand, there may be other learning opportunities related to the [Gordon and Gordon (1989)](#) study. First, the difference in sample means can be emphasized as a sample estimate for $\mu_1 - \mu_2$. The students can see how the difference in sample means changes from sample to sample, and thus, the concepts of estimation and random variables can be clearly illustrated. Second, the simulation experience may allow students to gain a better understanding of the concept of an estimated standard error of the difference in sample means. The concepts of sample estimates, random variables, and estimated standard errors can be especially difficult for introductory students to comprehend.

2.3 Confidence Intervals

Kennedy, Olinsky, and Schumacher (1990) suggested using simulation methods to illustrate the idea of inference and sampling error using MINITAB®. The class as a whole can be given a finite population for which they can calculate the mean and standard deviation. From this population, each student can generate a sample to find a 95% confidence interval for the population mean based on their sample mean. As an illustration of the interpretation of confidence intervals, the class can determine how many of the sample confidence intervals actually contain the true population mean.

An instructional module and corresponding software program developed by Marasinghe et al. (1996) also utilized simulation methods for the study of confidence intervals. This module and software program emphasized that confidence intervals vary from sample to sample, an understanding of the precision of a confidence interval, what is meant by the phrase "95% confident," and how sample size and confidence level affect confidence intervals.

Finally, Hesterberg (1998) reported that simulation methods can offer students intuitive understanding of confidence intervals (and other topics) through direct experiences and recommends an interactive language, particularly S-PLUS®, due to its flexibility. A display of 30 confidence intervals indicated by horizontal lines with the true mean shown as a vertical line is an effective way to visualize the notion of confidence. Horizontal lines that intersect the vertical line would indicate a confidence interval that contains the true mean.

Because confidence intervals can be generated for all statistics and are increasingly becoming more popular in the literature than hypothesis testing results (consider probability value versus an interval of values, as discussed in Pedazhur 1997), a more intuitive understanding of this concept can provide important long-term benefits. Simulation methods appear to be especially helpful for illustrating the interpretation and the "behavior" of confidence intervals (that is, whether the interval encloses the true parameter or not) and the "randomness" of the sample mean. Both are important concepts that students can generalize to other topics in statistics (for example, generating an interval estimate for population proportion, slope, or difference in means). In addition, the previous studies illustrate components of the constructivist learning approach. For example, the Kennedy et al. (1990) study involved a classroom of students interchanging ideas in hopes to foster reflection and identify misconceptions. Employing the use of more user-friendly commercial instructional software may also allow the teacher to assume more of a facilitator role, another important application of constructivism. Constructivist beliefs also require students to discover and develop meaning and understanding as independent and active learners, and using appropriate software, as suggested by Hesterberg (1998), is an important consideration in this learning process.

2.4 Binomial Distribution

The binomial distribution is an important discrete probability distribution that students in introductory statistics courses may encounter. Shibli (1990) reported that students never seem to explore the features of the binomial variable but instead focus on calculating probabilities. For this reason, he conducted a two-stage study on the binomial distribution where the first stage involved the familiar coin-tossing experiment (that is, physical simulation). All students are asked to flip a coin 10 times and record the number of heads (successes), x. If all the results are combined for the entire class, a relative frequency distribution of x might reveal that the relative frequencies of 4, 5, and 6 are usually greater than any other values. A potential problem can occur because values for the random variable $x = 0, 1, 9, 10$ may almost never occur, even though the students are told that the random variable x can assume values of $x = 0, 1, ..., 10$. Another potential caveat is that the relative frequency distribution from the classroom data may not resemble the theoretical shape of the probability distribution for $n = 10$ and $p = 0.5$ (the theoretical

distribution is approximately normal). With these problems in mind, Shibli (1990) proceeded to the second stage of the study (that is, computer simulation exercise) by using the RANDOM command in MINITAB®. A histogram can be constructed for $n = 10$, $p = 0.5$ for 30, 100, 1000, and 10000 repetitions. Increasing the repetitions may allow the students to see that the random variable x can assume values from 0 to 10 in addition to revealing the shape of each distribution. The program can also be modified to generate samples for different values of p to explore negative and positive skewness. Shibli (1990) concluded that students may benefit from being able to distinguish among the number of trials, n, and the number of times the experiment was repeated. Another advantage is that students can see that the binomial probabilities are the relative frequencies when the experiment is repeated a large number of times.

Others have advocated teaching the properties of the binomial distribution using simulation methods (that is, Pulley and Dolbear 1984; Goodman 1986; Hubbard 1992; Ricketts and Berry 1994). As shown in the Shibli (1990) study, these constructivst methods help students to understand facts related to the binomial distribution for themselves through their active participation and learning, instead of simply copying or receiving information conveyed by the teacher. Because the binomial distribution is the most important discrete probability distribution usually covered in introductory statistics courses, the concepts explored by Shibli (1990) may be worth the additional instructional time and effort because students will encounter similar concepts later in the course. Repeating the experiment a large number of times and varying the values of p foreshadow hypothesis testing and confidence intervals, which may provide an easier transition when these topics are subsequently covered in the context of estimating a population proportion. In addition, changing the values of p and sample size can further illustrate the CLT. Whether or not simulation methods versus a more traditional style of lecture promotes learning and is more advantageous for students is still a question that needs further investigation.

2.5 Regression Analysis

Regression analysis is another topic often taught in introductory statistics courses. Franklin (1992) suggested using simulation to illustrate some basic aspects of the simple linear regression model using MINITAB®. The simple linear regression model is given by: $y = \beta_0 + \beta_1 x + \varepsilon$ where y is the outcome or criterion, β_0 is the population intercept, β_1 is the population slope, x is the predictor variable, and ε is the random error. The class can choose a deterministic true model given by $y = \beta_0 + \beta_1 x$ relating two variables. The class can then choose values of x and their corresponding deterministic y values. Using MINITAB®, the class can simulate the random errors by drawing a random sample from a standard normal population. These random errors can then be added to each deterministic value of y to give $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$ The class can then be told that these are the actual y-values observed in practice. Next, the least squares line using the observed data may be computed and at this point students can clearly distinguish between the sample statistics b_0 and b_1 and the population parameters β_0 and β_1. Other concepts related to simple regression can also be explored, including hypothesis testing. For example, for the test of the hypothesis H_0: $\beta_1 = 0$ versus H_1: $\beta_1 \neq 0$, the population parameters are already known (that is, true model is already specified), and the resulting probability value from the hypothesis test can solidify concepts such as Type I error. In addition, the class can also construct a 95% confidence interval for β_1 and the observed data can be used to determine if in fact the true parameter, β_1, is captured within the interval.

Franklin (1992) also suggested using the same true linear model along with the same sample size and the same values of x, along with a new set of randomly generated errors from the standard normal distribution. This model helps to make the point that different values for b_0 and b_1 can be observed from

the same true line, same sample size, and same values of x. Thus, b_0 and b_1 can be seen as random variables and the purpose of hypothesis testing and constructing confidence intervals can be emphasized. Finally, two other illustrations can demonstrate first how spreading out the values of x can decrease the variance of the errors, and second how different values for the variance of the random error (that is, $\sigma = 3$ versus $\sigma = 1$) can result in different values for b_0 and b_1. These effects can ultimately affect the decision-making process. These and other regression-related concepts have been demonstrated using simulation methods (for example, the behavior of regression lines, the effects of changing b_0 and b_1, exploring the correlation coefficient or coefficient of determination) by many other researchers (see Jensen 1983; Pulley and Dolbear 1984; Olinsky and Schumacher 1990; Ferrall 1995; Romeu 1995; Marasinghe et al. 1996; Tryfos 1999).

Simulating random errors for the regression model to investigate their effect on the model is a very unique approach to teaching regression. The notion of random variables, sample estimates versus population parameters, Type I and II errors (power can also be considered), and confidence intervals for parameters are all abstract concepts that require additional instructional attention. Using CSMs to illustrate many of these concepts may encourage students to identify their misconceptions and change them based on their own construction of the concepts. In order to teach regression to beginning students using this approach, however, students should be at least familiar with concepts related to regression including correlation, random errors, probabilistic and deterministic models, interpretation of parameters, hypothesis testing, and the assumptions of regression analysis. In addition, determining values for σ may also be difficult for introductory students to put into perspective. Simulating random errors for a deterministic model and their effect on the model may not be very revealing for introductory students unless students have been exposed to some preliminary concepts. Perhaps this type of simulation exercise would be more useful in a more advanced course. Will the additional time it takes to implement this approach in the classroom or even as a computer project to solidify the concepts of regression as opposed to a more traditional approach serve to be advantageous for students? Empirical research in this area is also needed to answer this and other questions.

2.6 Sampling Distributions

Using CSMs to explore the properties of any sampling distribution is another way in which students can become active in the learning of important statistical concepts (Weir, McManus, and Kiely 1990; Marasinghe et al. 1996; delMas et al. 1999). The *Sampling Distribution* program was used by delMas et al. (1999) in their empirical study to help students gain a better understanding of sampling distributions and the CLT. This program allows students to construct their own understanding of sampling distributions and the CLT by specifying and changing the shape of a population, choosing different sample sizes, and exploring sampling distributions by randomly drawing large numbers of samples.

Students enrolled in an introductory statistics course (across different institutions) were expected to have read a chapter on sampling distributions and the CLT as well as participated in other simulation activities prior to using the *Sampling Distribution* program. In addition, class discussions before exposure to the program also focused on important concepts about sampling distributions and the CLT.

Considering a conceptual change model approach, the authors asked students to explore different scenarios using the program and then respond to a pre- and post-test. Overall, they found that although computer simulation methods can enrich a student's learning experience, additional activities are required in order for students to change their misconceptions. These additional activities support the conceptual change literature, which calls for supplemental exercises that require students to confront their misconceptions. Evaluating differences between students' own beliefs about chance events and the actual empirical results by allowing them to make predictions and test them, in addition to using the simulation

program, may help students obtain a clearer understanding of the concepts.

Another empirical study involved evaluating the teaching of sampling distributions using Monte Carlo simulations (Weir et al. 1990). A sample of 39 psychology students, matched on their grades in their first year statistics course, were exposed to simulations on concepts either related to the standard error of the mean or the F-distribution in an ANOVA context (consider the sampling distribution for the sample mean and the F statistic). For the two topics, demonstrations included sampling from a normal distribution, which was animated graphically on a window display followed by a generation of relevant statistics for the samples. Students in both demonstrations chose to run either one sample at a time when simulating or a specified number of samples simultaneously. For the standard error of the mean demonstration (*SEMDEMO*), the mean and variance were computed and for the F-distribution demonstration (*FDEMO*), the mean, standard deviation, sums of squares, and F-statistic were generated. Students were also able to select values for some parameters at the outset such as sample size for the *SEMDEMO*, or the relative difference between means and standard deviations for the *FDEMO*. Students in the *SEMDEMO* group (n = 20) observed how an increase in sample size would affect the value of the standard error of the mean, while students in the *FDEMO* group (n = 19) observed the value of the F-statistic for samples with equal means (that is, $F = 1.0$, where the F statistic is the ratio of estimated treatment effects and error to the estimated error variance). Students in the *FDEMO* group also experimented with the differences in means in standard deviation units, and the effect of varying sample variances on the F-statistic.

The interactive sessions lasted about an hour and were followed by exposure to a Monte Carlo demonstration during a lecture. Students were assessed using open-ended questions on a routine course test which included questions related to both demonstrations. If attainment of concepts was specific to the type of interaction, then students in the *FDEMO* group should perform better on questions related to their experience and students in the *SEMDEMO* group should do likewise. The results revealed that after students were categorized (according to their previous statistics grades) to a "high ability" or a "low ability" group, only lower ability students using the *FDEMO* showed greater improvement on the F-distribution questions relative to the scores of the lower ability students in the *SEMDEMO* group. After revising the *SEMDEMO* using student feedback, a second study revealed higher achievement for the lower ability students for both the *SEMDEMO* and *FDEMO* groups.

Introducing students to the concept of a sampling distribution using CSMs, based on a constructivist learning approach, is a powerful technique, which can provide greater insight and a more thorough understanding of statistics and their distributions. Most student do not realize that every statistic has a sampling distribution and using CSMs can provide a concrete way to illustrate this as well as a way to reveal how other factors, such as sample size, affect the sampling distribution. Sampling distributions can be generated for many commonly used statistics (such as the sample sum, mean, median, standard deviation, variance, range, and t-statistic). Visualizations can illustrate many of the properties mentioned previously including the facts that different statistics have different sampling distributions that depend on the specific statistic, sample size, and the parent distribution, that the variability in the sampling distribution can be decreased by increasing the sample size, and that for large samples, the sampling distribution can be approximated by a normal distribution. Many others have advocated using CSMs to teach sampling distributions (see Arnholt 1997; Schwarz and Sutherland 1997; Hesterberg 1998).

There are very few studies in the literature that empirically examine the impact of CSMs on student learning. The Weir et al. 1990 study revealed improved academic performance for lower-performing students only. The authors concluded that the reason for the improved performance was due to increased practice and deeper processing of concepts, due to the simulation exercises. According to the authors, the students' active involvement encouraged them to encode, remember, or structure concepts, which resulted in better problem-solving skills. delMas et al. (1999) may agree, but they would also advise that additional activities that promote conceptual change are also needed to facilitate this learning process.

2.7 Hypothesis Testing

Flusser and Hanna (1991) suggested using CSMs to study hypothesis testing, including exploring concepts such as power and Type I and Type II errors. Hypothesis testing can be studied considering an example of an unbalanced "valuable" dime with the probability of a head $P(H) = 0.75$ instead of $P(H) = 0.50$. Because of the uniqueness of this coin, one can decide whether dimes were valuable or ordinary by abiding by the following decision rule: If the coin is tossed 10 times and seven or more heads are obtained, the coin is valuable, otherwise the coin is ordinary. The authors suggested testing the following null hypothesis:

H_0: My coin is valuable ($P(H) = 0.75$)
H_1: My coin is ordinary ($P(H) = 0.50$)

(This author would reverse the above hypotheses to denote: with sufficient evidence, the null hypothesis that a coin is **ordinary** will be rejected). Using CSMs, the authors suggested observing over a large number of repetitions the empirical values of α, β, and Power $= 1 - \beta$. The point also can be emphasized that α and β behave like conditional probabilities, and thus both errors can not occur at the same time.

Hypothesis testing is one of the most difficult topics to teach, and a difficult topic for students to understand, especially introductory students. Flusser and Hanna (1991) wrote computer simulation programs (the software language was not reported) to help students gain a better understanding of power, Type I, and Type II errors. These concepts are indeed abstract and have little meaning for students when written in a textbook. Here is one example where CSMs can again offer an alternative teaching technique using the constructivist learning approach. The key is to help learners to understand alternative points of view and resolve conflicts among incompatible solution methods. Other researchers have advocated using these methods to teach hypothesis testing and Type I and Type II errors (see Taylor and Bosch 1990; Jockel 1991; Bradley et al. 1992; Kleiner and Borenstein 1993; Ricketts and Berry 1994; Arnholt 1997).

2.8 Survey Sampling

Simulation methods can also be used to teach introductory or advanced concepts in a sample survey course. Chang, Lohr, and McLaren (1992) examined a simulation program called "*SURVEY*" that simulates samples from a fictitious county by providing data related to cities, municipalities, districts, houses, and rural or urban areas. This program allows students to become involved in all stages of the sampling process, from designing and analyzing the survey to dealing with nonresponse.

Students using *SURVEY* can be given prior information and conjectures about the composition and homogeneity of the population. The students can then use the information provided to design surveys and learn which sampling schemes (simple, cluster, stratified, or quota) and estimators would be the most efficient in different parts of the county.

The authors reported many academic benefits at the end of the term for their students, but there was no mention of an empirical study conducted using this program. In particular, they reported that their students were more concerned with "getting the correct sampling scheme" as opposed to "getting the right answer in the back of the book." The authors also stated that the students seemed to enjoy using the *SURVEY* program.

The *SURVEY* program can be a very valuable teaching tool because it allows students to become involved with real sampling problems and thus, force them to think about every aspect of the surveying

process. The students may realize that some ideas are not so easily carried out in practice. In addition, constructivism would suggest that real-life problems tend to invite more discussion and interaction among students, facilitating learners to become independent thinkers. One disadvantage of using this program, however, may be that students need to use an additional software package to analyze the data, which must include logical and subset selection capabilities, according to the authors. Even though logical and subset capabilities are available in most user-friendly packages, students must have the skills to write these programs, especially when statistics must be calculated from cluster or stratified samples. In this case, statistics are needed for each cluster or stratum, which can be very time consuming if students are not very familiar with statistical operations. More advanced sampling schemes can require more advanced data programming which can limit introductory students' understanding or require much more classroom time. Many sampling classes do not require an advanced knowledge of statistical programming. As an alternative, this program may be more useful and practical for advanced-level students, or an instructor familiar with writing programs or macros can supplement the program by implementing the necessary operations for introductory students.

No additional programming is needed for a DOS-based computer simulation program called "*SAMPLE*" developed by Kalsbeek (1996). The program is a first step of a long range plan of useful teaching tools that empirically displays important principles of sampling, emphasizing simple random, stratified simple random, and cluster sampling. Another sampling program called *StatVillage* uses a World Wide Web-based interface that allows students to use real data from a census file to illustrate sampling schemes and concepts (Schwarz 1997). The author reported that a standard platform using any browser as well as exposing students to actual census data may provide for a better learning experience and appreciation of how to effectively deal with data imperfections. Other researchers have experimented with using CSMs to teach important sampling concepts (see Horgan 1991; Fecso et al. 1996).

3. Summary

The literature reviewed in this article brings the following issues to light. First, CSMs are being used in all areas of statistics to help students understand difficult concepts, from mathematics and education to business and medicine. In addition, a variety of different topics are also being considered, from introductory concepts, which emphasize frequency histograms and the CLT, to more advanced topics such as Bayesian methods, time series, and ANCOVA. Second, although many of the authors recommended using CSMs as a teaching tool in the statistics classroom, a few also advocated using some type of simulation exercise prior to the use of CSMs. The overall consensus was that CSMs (either with physical simulation or without) appeared to facilitate student understanding of difficult or abstract concepts. Very few suggested that CSMs are not effective or that students don't understand the computer simulation results (see Kaigh 1996; Velleman and Moore 1996). Third, although the need for novel and interesting instructional methods to improve student achievement in statistics is forever warranted, one major disadvantage evident in the literature was the lack of empirical and theoretical research and support used to substantiate the recommendations. Using many of the innovative ideas described in this review may be beneficial to students academically, but these methods must be evaluated, documented empirically, and rest on the foundation of a specific learning theory, particularly if claims are made that student achievement is enhanced. The results from the empirical studies reviewed in this paper revealed that CSMs appeared to be effective for lower-ability students. Also, learning appears to be enhanced when using CSMs also involves exercises where students are able to confront their faulty ideas or misconceptions.

The lack of empirical support may be related to the fact that it can be very difficult and labor-intensive to conduct empirical or experimental educational research. Randomly assigning students in the same class to different teaching methods is also an ethical concern. Still, there is a growing emphasis on assessment in higher and statistics education (Garfield 1995), especially using computer technology, and

documenting our research through empirical methods will be necessary as we progress into the computer education era.

Teachers of statistics are always searching for new or alternative teaching methods to improve statistics instruction in hopes of enhancing student learning and to improve student attitudes toward statistics. The numerous recommendations to offer CSMs as a teaching and learning tool are clear examples. CSMs offer students the opportunity for unique and concrete learning experiences where an individual construction of meaning and ideas about statistics concepts is obtained. Considering the literature reviewed regarding CSMs and the previously mentioned issues, future research might consider investigation of physical versus computer simulation methods to determine what impact either or both of these methods have on student learning, as well as whether or not one method is superior to another. Also, because there are very few empirical studies, additional research is needed in all of the topics reviewed to provide statistics educators with statistical and theoretical evidence of the advantages, if any, of these methods. With the rapid advancements in technology and as today's statistics classroom environments continue to embrace the Internet and Web-based learning, empirical research that documents student performance will continue to be essential.

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4. Appendix - Summary of CSM Literature Review

Authors (Date)	Topic	Target Audience	Empirical Study?
Albert (1993)	Bayesian statistics	IT	no
Arnholt (1997)	Sampling distribution; Statistical power; Statistical efficiency	IT, IN, A	no
Bradley, Hemstreet, and Ziergenhagen (1992)	Sampling distribution; CLT; Type I and Type II errors; Power; Violation of assumptions; Orthogonal contrast	IT, IN, A	no
Braun (1995)	Contingency table; Elementary probability	IN, A	no
Chang, Lohr, and McLaren (1992)	Survey sampling	IT	no
Dambolena (1986)	CLT; Student's t-distribution	IT	no
delMas, Garfield, and Chance (1999)	Sampling distributions; CLT	IT	yes
Fecso, et al. (1996)	Survey sampling	IT, IN	no
Ferrall (1995)	Econometrics; Regression analysis	IT, IN	no
Flusser and Hanna (1991)	Hypothesis testing; Type I and Type II errors	IT	no
Franklin (1992)	Linear regression	IT	no
Goodman (1986)	Basic statistics concepts	IT	no
Gordon and Gordon (1989)	Sampling distribution	IT	no
Halley (1991)	Survey sampling	IT	no
Hesterberg (1998)	Sampling distribution; CLT; Confidence interval; Cauchy distributions; Regression methods; Bayesian methods	IT, IN, A	no

Horgan (1991)	Survey sampling	IT, IN	no
Hubbard (1992)	Binomial distribution	IT	no
Jockel (1991)	Hypothesis testing	IT, IN	no
Kader (1991)	Probability	IT	no
Kaigh (1996)	Regression analysis	IT	no
Kalsbeek (1996)	Survey sampling	IT, IN, A	no
Karley (1990)	Frequency histograms; CLT; Tests of normality	IT	no
Kennedy, Olinksy, and Schumacher (1990)	CLT; Confidence interval; Regression analysis; Bootstrapping	IT, IN, A	no
Kersten (1983)	Sampling distribution; CLT	IT	no
Kleiner and Borenstein (1993)	Power analysis	IT, IN	no
Marasinghe, et al. (1996)	Graphical displays; Departures from linearity; Confidence interval; CLT	IT	no
Mittag(1992)	CLT	IT	no
Ng and Wong (1999)	Sampling distribution; CLT; Law of Large Numbers	IT	no
Perry and Kader (1995)	Estimation	IT	no
Pickover (1991)	Randomness	IT, IN	no
Pulley and Dolbear (1984)	Economics statistics	IT, IN, A	no
Reich and Arvanitis (1991)	Sampling	IT	no
Ricketts and Berry (1994)	Hypothesis testing; t-distribution; Binomial distribution; Confidence limits	IT, IN, A	no
Romeu (1995)	ANOVA; ANCOVA; Multiple regression; Goodness-of-fit; Residual analysis; Variable selection; Response surface methodology	IN, A	no
Schwarz (1997)	Survey sampling	IT, IN	no
Schwarz and Sutherland (1997)	Sampling distribution	IT	no
Shibli (1990)	Binomial distribution	IT	no
Simon and Bruce (1991)	Probability and statistics	IT, IN, A	no
Stent and McAlevey (1991)	CLT	IT	no
Stirling (1991)	Measures of variability and central tendency; CLT; Correlation and regression; Confidence interval; Hypothesis testing	IT	no
Taylor and Bosch (1990)	Logistic regression; Type I error; Power; Hypothesis testing	IT, IN, A	no
Tijms (1991)	Law of Large Numbers; Random walk; CLT	IT	no
Tryfos (1999)	Data analysis; Regression; Time series	IN, A	no
Velleman and Moore (1996)	Randomness; Probability; Inference	IT, IN	no
Wallgren and Girardeau	Statistical theory	A	no

(1993)			
Weir, McManus, and Kiely (1990)	Sampling distribution	IT	yes
West and Ogden (1998)	CLT; Confidence interval; Histogram; Influential point	IT, IN	no
Yu, Behrens, and Anthony (1995)	CLT	IT	no

IT = introductory, IN = intermediate, A = advanced

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